EXAM 2 REVIEW KEY

Note Title 10/14/2012

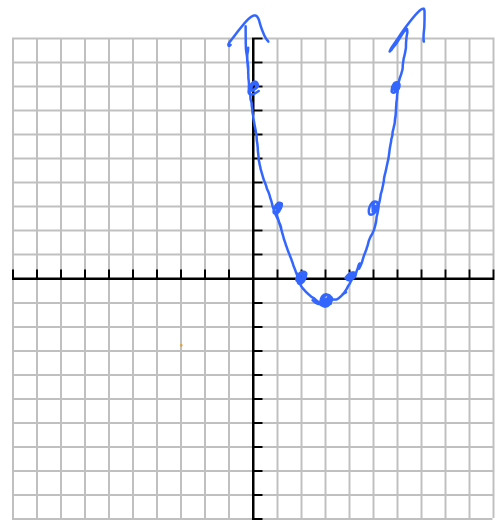
1. $f(x)=x^2-6x+8$ $\qquad -\frac{6}{2}=-3$

$f(x)=(x^2-6x\qquad)+8$ $\qquad (-3)^2=9$

$f(x)=(x^2-6x+9)+8-9$

$f(x)=(x-3)^2-1$

$$\boxed{\begin{array}{c}\text{vertex: } (3,-1)\\ \text{axis of symmetry: } x=3\end{array}}$$



2. $f(x)=-x^2+4x$ $\qquad \frac{-4}{2}=-2$

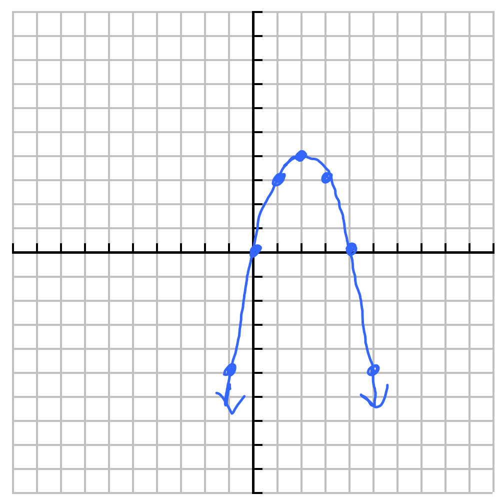
$f(x)=-(x^2-4x\qquad)$ $\qquad (-2)^2=4$

$f(x)=-\underbrace{(x^2-4x+4)}_{-4}+4$

added zero

$f(x)=-(x-2)^2+4$

$$\boxed{\begin{array}{c}\text{vertex: } (2,4)\\ \text{axis of symmetry: } x=2\end{array}}$$



3. $f(x)=3x^2+6x-7$ $\qquad \frac{2}{2}=1$

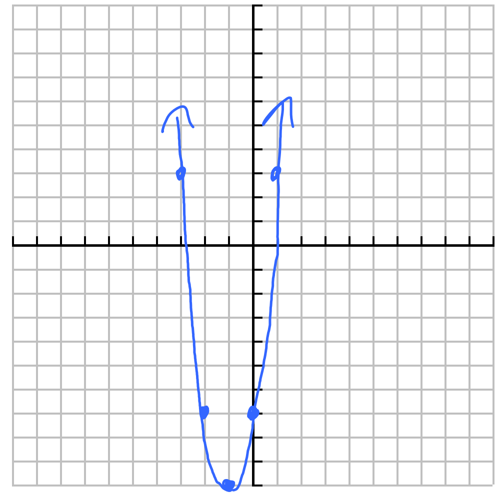
$f(x)=3(x^2+2x\qquad)-7$ $\qquad 1^2=1$

$f(x)=3\underbrace{(x^2+2x+1)}_{+3}-7-3$

added zero

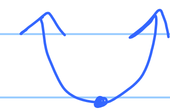
$f(x)=3(x+1)^2-10$

$$\boxed{\begin{array}{c}\text{vertex: } (-1,-10)\\ \text{axis of symmetry: } x=-1\end{array}}$$



4. $f(x) = 2x^2 + 5x - 3$

opens up, so it has a minimum.



$$x = \frac{-b}{2a} = \frac{-5}{2(2)} = \frac{-5}{4}$$

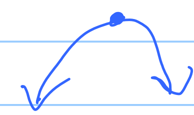
$$f\left(-\tfrac{5}{4}\right) = 2\left(-\tfrac{5}{4}\right)^2 + 5\left(-\tfrac{5}{4}\right) - 3 = -\frac{49}{8}$$

vertex: $\left(-\frac{5}{4}, -\frac{49}{8}\right)$

$\boxed{\text{minimum value} = -\frac{49}{8}}$

5. $f(x) = -x^2 - 4x + 3$

opens down, so it has a maximum.



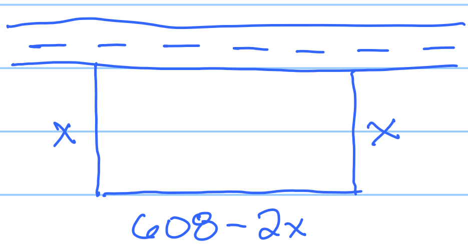
$$x = \frac{-b}{2a} = \frac{4}{2(-1)} = \frac{4}{-2} = -2$$

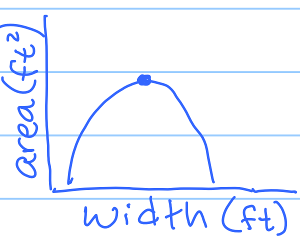
$$f(-2) = -(-2)^2 - 4(-2) + 3 = 7$$

vertex: $(-2, 7)$

$\boxed{\text{maximum value} = 7}$

6.





$$A(x) = x(608 - 2x) = 608x - 2x^2$$

$$A(x) = -2x^2 + 608x$$

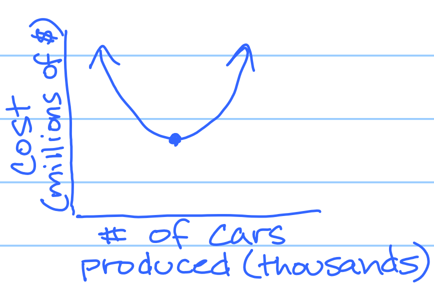
$$x = \frac{-b}{2a} = \frac{-608}{2(-2)} = 152 \text{ ft}$$

$$A(152) = -2(152)^2 + 608(152)$$

$$= \boxed{46{,}208 \text{ ft}^2}$$

The largest area that can be enclosed is 46,208 ft². This occurs when the width is 152 ft.

7. $C(x) = 3x^2 - 18x + 63$



$$x = \frac{-b}{2a} = \frac{18}{2(3)} = 3$$

The company must produce 3000 cars to minimize the cost.

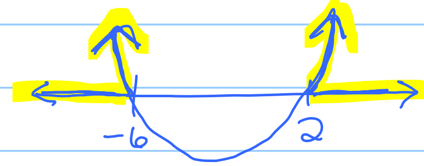
8. $x^2+4x>12$

$x^2+4x-12>0$

$(x+6)(x-2)>0$

x-ints: $-6, 2$

opens up



Want to know where $y>0$.
Where is the graph above the x-axis?

$\boxed{(-\infty,-6)\cup(2,\infty) \text{ or } \{x \mid x<-6 \text{ or } x>2\}}$

9. $2x^2+5x-3\le 0$

$(2x^2+6x)+(-x-3)\le 0$

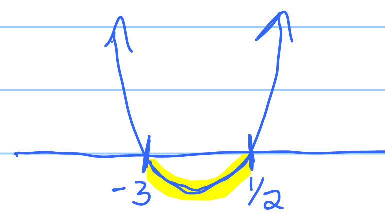
$2x(x+3)-1(x+3)\le 0$

$(x+3)(2x-1)\le 0$

x-ints: $-3, \frac{1}{2}$

opens up

$2(-3)=-6$	5
6, -1	5



Want to know where $y\le 0$.
Where is graph on or below the x-axis?

$\boxed{[-3,\frac{1}{2}] \text{ or } \{x \mid -3\le x\le \frac{1}{2}\}}$

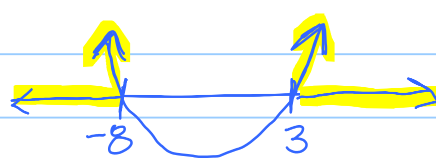
10. $g(x)=x^2+5x-24$ $\quad g(x)\ge 0$

$x^2+5x-24\ge 0$

$(x+8)(x-3)\ge 0$

x-ints: $-8, 3$

opens up



Want to know where $y\ge 0$.
Where is graph on or above the x-axis?

$\boxed{(-\infty,-8]\cup[3,\infty) \text{ or } \{x \mid x\le -8 \text{ or } x\ge 3\}}$

11. $f(x) = 2x^2(x-5)(x+3)^3$

zeros: 0 mult. 2 touches; 5 mult. 1 crosses; -3 mult. 3 crosses

12. $f(x) = (x-3)^2(x^2+4)$

zeros: 3 mult. 2 touches; (x^2+4): no real zeros

13. $f(x) = -3(x+7)(x-2)^2$

End behavior: $y = -3x(x^2)$

$\boxed{y = -3x^3}$ $\uparrow$ $\downarrow$

Max turning pts = degree -1 = $\boxed{2}$

14. $f(x) = x^3(x+4)(x^2+5)$

End behavior: $y = x^3(x)(x^2)$

$\boxed{y = x^6}$ $\uparrow$ $\uparrow$

Max turning pts $= 6-1 = \boxed{5}$

15. $f(x) = (x+2)^3(x-1)^2(x+1)$

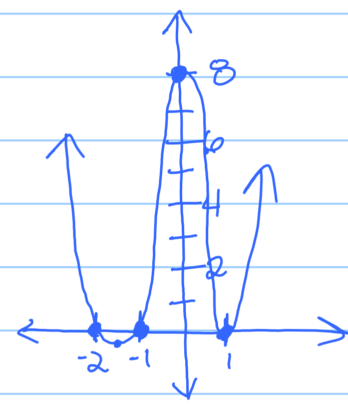
a&b) x-ints: -2 mult. 3 crosses; 1 mult 2 touches; -1 mult 1 crosses

y-int: $f(0) = (0+2)^3(0-1)^2(0+1) = 8$

c) End behavior: $y = (x^3)(x^2)(x)$

$y = x^6 \leftarrow$ Both ends up

d) Max turning pts: $6-1 = 5$



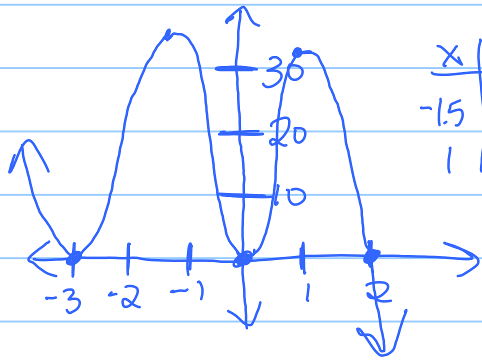
16. $f(x) = -2x^2(x-2)(x+3)^2$

a & b) x-ints: 0 mult 2 touches; 2 mult 1 crosses; -3 mult 2 touches

y-int: $f(0) = -2(0)^2(0-2)(0+3)^2 = 0$

c) End behavior: $y = -2x^5$ ↖ ↘

d) Max turning pts: $5-1=4$



x	y
-1.5	35.4
1	32

17. $R(x) = \dfrac{x-5}{x^2+3x+2} = \dfrac{x-5}{(x+2)(x+1)}$

Domain: $(x+2)(x+1) \neq 0$

$\boxed{\{x \mid x \neq -2, -1\}}$

18. $R(x) = \dfrac{x^2+3x}{x^2-x-12} = \dfrac{x(x+3)}{(x-4)(x+3)}$

Find domain before simplifying.

Domain: $(x-4)(x+3) \neq 0$

$\boxed{\{x \mid x \neq 4, -3\}}$

19. $R(x) = \dfrac{7}{x^2+3x-40} = \dfrac{7}{(x+8)(x-5)}$

Vertical asymptotes: Simplify if possible, then set denominator = 0.

$(x+8)(x-5) = 0$

$\boxed{x = -8 \ \& \ x = 5}$ ← Must have the "x=". Asymptotes are lines. You are giving the equation of the line.

20. $R(x) = \dfrac{-x^2+4x-4}{x^2-5x+6} = \dfrac{-(x^2-4x+4)}{x^2-5x+6} = \dfrac{-(\cancel{x-2})(x-2)}{(\cancel{x-2})(x-3)}$

hole @ $x=2$

vertical asymptote: $x-3=0$

$\boxed{x=3}$

21. $R(x) = \dfrac{20x}{x+12}$ $\qquad \dfrac{\text{Deg } 1}{\text{Deg } 1} \Rightarrow$ $\boxed{\text{horizontal asymptote.}}$

Make fraction out of leading coefficients.

$y = \dfrac{20}{1}$

$\boxed{y=20}$

22. $R(x) = \dfrac{4x^2-3x+7}{x-2}$ $\qquad \dfrac{\text{Deg } 2}{\text{Deg } 1} \Rightarrow$ $\boxed{\text{oblique asymptote}}$

Use long division, or, in this case, synthetic division also works. (Yay!)

$$\begin{array}{r|rrr} 2 & 4 & -3 & 7 \\ & & 8 & 10 \\ \hline & 4 & 5 & 17 \end{array}$$

$\boxed{y=4x+5}$

or

$$\begin{array}{r} 4x+5 \\ x-2\,\overline{)\,4x^2-3x+7} \\ -(4x^2-8x) \\ \hline 5x+7 \\ -(5x-10) \\ \hline 17 \end{array}$$

23. $R(x) = \dfrac{x-5}{x^2+3}$ $\qquad \dfrac{\text{Deg } 1}{\text{Deg } 2}$

If deg. numerator < deg. denom, there is a $\boxed{\text{horizontal asymptote}}$ at $\boxed{y=0}$.

24. $R(x) = \dfrac{x}{x^2-25} = \dfrac{x}{(x+5)(x-5)}$

Domain: $(x+5)(x-5) \neq 0$ $\boxed{\{x \mid x \neq -5, 5\}}$

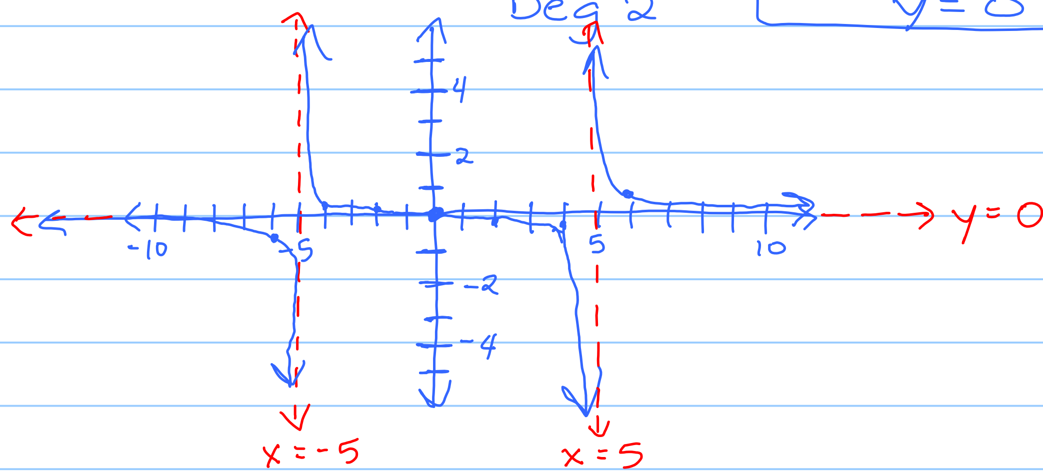
x-int: $x = 0$ $\boxed{(0,0)}$

y-int: $R(0) = \dfrac{0}{0^2-25} = 0$ $\boxed{(0,0)}$

Vertical asymptotes: $(x+5)(x-5)=0$ $\boxed{x=-5,\ x=5}$

horiz/oblique asymptote: $\dfrac{\text{Deg } 1}{\text{Deg } 2} \Rightarrow$ $\boxed{\text{horiz asymp: } y=0}$

x	y
-6	-.5
-4	.4
-2	.1
2	-.1
4	-.4
6	.5



25. $R(x) = \dfrac{x^2+5x-6}{x+2} = \dfrac{(x+6)(x-1)}{x+2}$

Domain: $x+2 \neq 0$ $\boxed{\{x \mid x \neq -2\}}$

x-ints: $(x+6)(x-1)=0$ $\boxed{(-6,0),(1,0)}$

y-int: $R(0) = \dfrac{0^2+5(0)-6}{0+2}$ $\boxed{(0,-3)}$

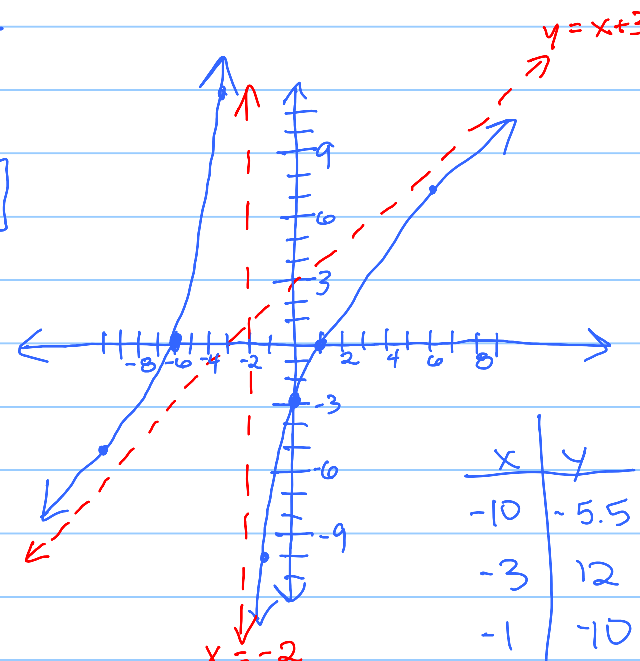
vert. asymp: $x+2=0$ $\boxed{x=-2}$

horiz/oblique: $\dfrac{\text{Deg } 2}{\text{Deg } 1} \Rightarrow$ $\boxed{\text{Oblique } y=x+3}$

$$\begin{array}{r}
x+3 \\
x+2 \overline{)\, x^2+5x-6} \\
-(x^2+2x) \\
\hline
3x-6 \\
-(3x+6) \\
\hline
-12
\end{array}$$

or

$$\begin{array}{r|rrr}
-2 & 1 & 5 & -6 \\
 & & -2 & -6 \\
\hline
 & 1 & 3 & -12
\end{array}$$



x	y
-10	-5.5
-3	12
-1	-10
6	7.5

26. $R(x) = \dfrac{x^2+3x}{x^2+2x-3} = \dfrac{x\cancel{(x+3)}}{(x-1)\cancel{(x+3)}} = \dfrac{x}{x-1}$

hole @ $x=-3$

Domain: $(x-1)(x+3) \neq 0$ $\boxed{\{x \mid x \neq 1, -3\}}$

x-int: $x = 0$ $\boxed{(0,0)}$

y-int: $R(0) = \dfrac{0}{0-1} = 0$ $\boxed{(0,0)}$

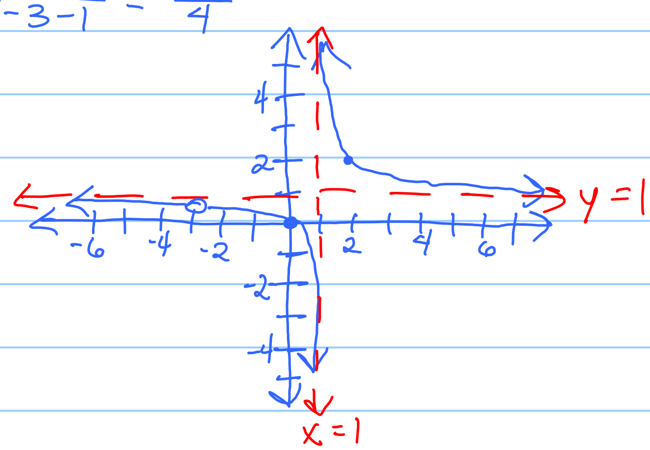
Vert. Asymp: $x - 1 = 0$ $\boxed{x=1}$

Horiz/Oblique Asymp: $\dfrac{\text{Deg } 2}{\text{Deg } 2}$ Ratio of leading coefficients: $y = \frac{1}{1}$

$\boxed{\text{Horiz. asymp: } y=1}$

Hole @ $x=-3$. Plug -3 into simplified function to find y-coord. $\boxed{\text{Hole: } (-3, \tfrac{3}{4})}$

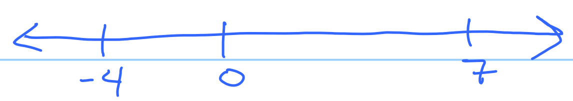
$\dfrac{-3}{-3-1} = \dfrac{3}{4}$



x	y
2	2

27. $\underbrace{x(x+4)(7-x)}_{} \leq 0$

This $= 0$ @ $0, -4, \& 7$



Interval:	$(-\infty, -4]$	$[-4, 0]$	$[0, 7]$	$[7, \infty)$
Pick an x:	-5	-1	1	8
f(x) :	60	-24	30	-96

$f(x) < 0$

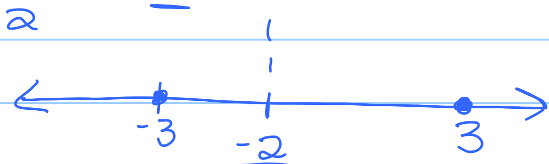
$\boxed{[-4, 0] \cup [7, \infty)}$

Use brackets because we want to include where $f(x) = 0$.

28. $\dfrac{x^2-9}{x+2} \geq 0$ $\qquad \dfrac{(x+3)(x-3)}{x+2} \geq 0$

x-ints: -3 & 3

undef @ $x=-2$



Interval:	$(-\infty,-3]$	$[-3,-2)$	$(-2,3]$	$[3,\infty)$
Pick an x:	-4	-2.5	0	4
$f(x)$:	-3.5	5.5	-4.5	1.2

$f(x) > 0$

$$\boxed{[-3,-2) \cup [3,\infty)}$$

Use brackets on the -3 & 3 because $f(x)=0$ there, but a parenthesis on the -2 because $f(x)$ is undefined there.

29. $\dfrac{x+2}{x-3} > 1$

$\dfrac{x+2}{x-3} - 1 > 0$

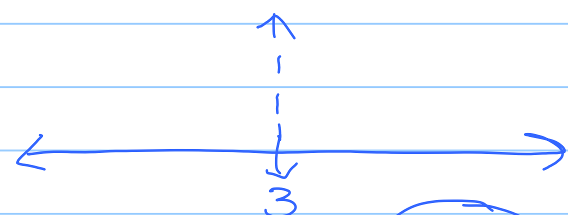
$\dfrac{x+2}{x-3} - 1\left(\dfrac{x-3}{x-3}\right) > 0$

$\dfrac{x+2-x+3}{x-3} > 0$

$\underbrace{\dfrac{5}{x-3}}_{} > 0$

no x-ints.

undef @ $x=3$



Interval:	$(-\infty,3)$	$(3,\infty)$
Pick an x:	0	4
$f(x)$:	$-\frac{5}{3}$	5

$$\boxed{(3,\infty)}$$

30. $(x^2-3x+8) \div (x+2)$

-2	1	-3	8
		-2	10
	1	-5	18

$$\boxed{x-5 \quad R18 \quad \text{or} \quad x-5+\frac{18}{x+2}}$$

31. $(3x^3 - x + 2) \div (x - 4)$

$$\begin{array}{r|rrrr} 4 & 3 & 0 & -1 & 2 \\ & & 12 & 48 & 188 \\ \hline & 3 & 12 & 47 & 190 \end{array}$$

$$\boxed{\begin{array}{l} 3x^2 + 12x + 47 \quad R\,190 \quad \text{or} \\ 3x^2 + 12x + 47 + \frac{190}{x-4} \end{array}}$$

32. Remainder when $f(x) = x^4 - 3x^3 + 2x + 1$ is divided by $x + 2$: Calculate $f(-2)$.

$$f(-2) = (-2)^4 - 3(-2)^3 + 2(-2) + 1$$
$$= 16 + 24 - 4 + 1 = \boxed{37}$$

33. $f(x) = -3x^3 + 5x^2 - 4x + 12$

Factors of 12: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Factors of -3: $\pm 1, \pm 3$

Potential rational zeros: $\boxed{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12, \pm 1/3, \pm 2/3, \pm 4/3}$

34. $f(x) = x^3 - 5x^2 + 2x + 8$

Factors of 8: $\pm 1, \pm 2, \pm 4, \pm 8$

Factors of 1: ± 1

Potential rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8$

Test on calculator:

$f(-1) = 0, \quad f(2) = 0, \quad f(4) = 0$

$$\boxed{\begin{array}{l} \text{Zeros: } -1, 2, \text{\& } 4 \\ f(x) = (x+1)(x-2)(x-4) \end{array}}$$

35. $f(x) = x^4 - 2x^3 + 6x^2 - 18x - 27$

Factors of -27: $\pm 1, \pm 3, \pm 9, \pm 27$

Factors of 1: ± 1

Potential rational zeros: $\pm 1, \pm 3, \pm 9, \pm 27$

$f(-1) = 0 \qquad f(3) = 0$

$\underline{-1}$	1	-2	6	-18	-27
		-1	3	-9	27
$\underline{3}$	1	-3	9	-27	0
		3	0	27	
	1	0	9	0	

$f(x) = (x+1)(x-3)(x^2+9)$ no real zeros

real zeros: $-1, 3$

36. $x^3 + 5x^2 - 2x - 10 = 0$

Factors of -10: $\pm 1, \pm 2, \pm 5, \pm 10$

Factors of 1: ± 1

Potential rational zeros: $\pm 1, \pm 2, \pm 5, \pm 10$

$f(-5) = 0$

$\underline{-5}$	1	5	-2	-10
		-5	0	10
	1	0	-2	0

$(x+5)(x^2-2) = 0$

$x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

$\boxed{\{-5, -\sqrt{2}, \sqrt{2}\}}$

37. $2x^4 + x^3 + x^2 + 41x - 21 = 0$

Factors of -21: $\pm 1, \pm 3, \pm 7, \pm 21$

Factors of 2: $\pm 1, \pm 2$

Potential rational zeros: $\pm 1, \pm 3, \pm 7, \pm 21, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{7}{2}, \pm \frac{21}{2}$

$f(-3) = 0 \qquad f(\tfrac{1}{2}) = 0$

$$\begin{array}{r|rrrrr}
-3 & 2 & 1 & 1 & 41 & -21 \\
 & & -6 & 15 & -48 & 21 \\
\hline
\tfrac{1}{2} & 2 & -5 & 16 & -7 & \multicolumn{1}{|r}{0} \\
 & & 1 & -2 & 7 & \\
\hline
 & 2 & -4 & 14 & \multicolumn{1}{|r}{0} &
\end{array}$$

$(x+3)(x-\tfrac{1}{2})(2x^2 - 4x + 14) = 0$

$2(x+3)(x-\tfrac{1}{2})(x^2 - 2x + 14) = 0$

Quadratic formula: $x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(7)}}{2(1)}$

$\boxed{\{-3, \tfrac{1}{2}\}}$ $\qquad x = \dfrac{2 \pm \sqrt{-24}}{2}$ ← not real

38. Degree: 4 $\qquad$ Zeros: $2-i,\ 4i$

Other zeros: $\boxed{2+i,\ -4i}$

39. Degree: 3 $\qquad$ zeros: $5,\ -2+3i$

Other zero: $\boxed{-2-3i}$

40. Degree: 4 $\qquad$ zeros: $-3, 1, 2-i$

Other zero: $2+i$

$f(x) = (x+3)(x-1)(x-(2-i))(x-(2+i))$

$f(x) = (x+3)(x-1)(x-2+i)(x-2-i)$

$f(x) = (x^2 + 2x - 3)(x^2 - 4x + 5)$

$\boxed{f(x) = x^4 - 2x^3 - 6x^2 + 22x - 15}$

	x	-2	$+i$
x	x^2	$-2x$	$+ix$
-2	$-2x$	$+4$	$-2i$
$-i$	$-ix$	$+2i$	$-i^2 = +1$

	x^2	$+2x$	-3
x^2	x^4	$+2x^3$	$-3x^2$
$-4x$	$-4x^3$	$-8x^2$	$+12x$
$+5$	$+5x^2$	$+10x$	-15

41. Degree: 5 zeros: $2,\ 3i,\ -1+4i$

other zeros: $-3i,\ -1-4i$

$f(x) = (x-2)(x-3i)(x+3i)(x-(-1+4i))(x-(-1-4i))$

$f(x) = (x-2)(x-3i)(x+3i)(x+1-4i)(x+1+4i)$

$f(x) = (x-2)(x^2+9)(x+1-4i)(x+1+4i)$

$f(x) = (x^3-2x^2+9x-18)(x^2-4x+5)$

$\boxed{f(x) = x^5 + 22x^3 - 34x^2 + 117x - 306}$

$(x-3i)(x+3i) =$

$x^2 + 3ix - 3ix - 9i^2 =$

$x^2 + 9$

	x	$+1$	$-4i$
x	x^2	$+x$	$-4ix$
$+1$	$+x$	$+1$	$-4i$
$+4i$	$+4ix$	$+4i$	$-16i^2 = 16$

$(x-2)(x^2+9) =$

$x^3 + 9x - 2x^2 - 18$

	x^3	$-2x^2$	$+9x$	-18
x^2	x^5	$-2x^4$	$+9x^3$	$-18x^2$
$+2x$	$+2x^4$	$-4x^3$	$+18x^2$	$-36x$
$+17$	$+17x^3$	$-34x^2$	$+153x$	-306

42. $f(x) = x^4 + 2x^2 - 63$ Zero: $3i \Rightarrow -3i$ is also a zero

$(x-3i)$ & $(x+3i)$ are factors

$(x-3i)(x+3i) = x^2+9$

$f(x) = (x^2+9)(\ ?\ )$

use long division

$$\begin{array}{r} x^2 - 7 \\ x^2+9 \overline{) x^4 + 2x^2 - 63} \\ -(x^4 + 9x^2) \\ \hline -7x^2 - 63 \\ -(-7x^2 - 63) \\ \hline 0 \end{array}$$

$f(x) = (x^2+9)(x^2-7)$

$x^2 - 7 = 0$

$x^2 = 7$ $x = \pm\sqrt{7}$

$\boxed{\text{additional zeros: } -3i, -\sqrt{7}, \sqrt{7}}$

43. $f(x) = x^3 - 3x^2 - 5x + 39$ zero: -3

$$\begin{array}{r|rrrr} -3 & 1 & -3 & -5 & 39 \\ & & -3 & 18 & -39 \\ \hline & 1 & -6 & 13 & \multicolumn{1}{|r}{0} \end{array}$$

$f(x) = (x+3)(x^2 - 6x + 13)$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(13)}}{2(1)} = \frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i$$

$\boxed{\text{zeros: } -3,\ 3+2i,\ 3-2i}$

44. $f(x) = 2x^4 + 3x^3 + 6x^2 + 12x - 8$

Factors of -8: $\pm 1, \pm 2, \pm 4, \pm 8$

Factors of 2: $\pm 1, \pm 2$

Potential rational zeros: $\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}$

$f(-2) = 0$ $\quad f(\tfrac{1}{2}) = 0$

$$\begin{array}{r|rrrrr} -2 & 2 & 3 & 6 & 12 & -8 \\ & & -4 & 2 & -16 & 8 \\ \hline \frac{1}{2} & 2 & -1 & 8 & -4 & \multicolumn{1}{|r}{0} \\ & & 1 & 0 & 4 & \\ \hline & 2 & 0 & 8 & \multicolumn{1}{|r}{0} & \end{array}$$

$f(x) = (x+2)(x - \tfrac{1}{2})(2x^2 + 8)$

$f(x) = 2(x+2)(x - \tfrac{1}{2})(x^2 + 4)$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm\sqrt{-4} = \pm 2i$

$$\boxed{\begin{array}{l} f(x) = 2(x+2)(x - \tfrac{1}{2})(x + 2i)(x - 2i) \\ \text{zeros: } -2,\ \tfrac{1}{2},\ -2i,\ 2i \end{array}}$$

45. $f(x) = x^3 + 11x^2 + 36x + 26$

Factors of 26: $\pm 1, \pm 2, \pm 13, \pm 26$

Factors of 1: ± 1

Potential rational zeros: $\pm 1, \pm 2, \pm 13, \pm 26$

$f(-1) = 0$

$$\begin{array}{r|rrrr} -1 & 1 & 11 & 36 & 26 \\ & & -1 & -10 & -26 \\ \hline & 1 & 10 & 26 & 0 \end{array}$$

$f(x) = (x+1)(x^2 + 10x + 26)$

$x = \dfrac{-10 \pm \sqrt{10^2 - 4(1)(26)}}{2(1)}$

$x = \dfrac{-10 \pm \sqrt{-4}}{2} = \dfrac{-10 \pm 2i}{2}$

$x = -5 \pm i$

$f(x) = (x+1)(x - (-5+i))(x - (-5-i))$

$$\boxed{\begin{array}{c} f(x) = (x+1)(x+5-i)(x+5+i) \\ \text{zeros: } -1, -5+i, -5-i \end{array}}$$